Mental Maths Strategies ...

A PRACTICAL EXPLANATION

By Paul Swan

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He was awarded his PhD for his research on children's computation choices and methods.

Curriculum documents throughout Australia are highlighting the need for children to develop mental strategies for performing calculations. In Western Australia, the Calculate substrand of the Number strand states that students should learn, choose and use a repertoire of mental, paper and calculator strategies, meeting needed levels of accuracy and judging the reasonableness of results. Likewise, the NSW curriculum refers to students selecting and using appropriate mental techniques to calculate. These statements raise the question:

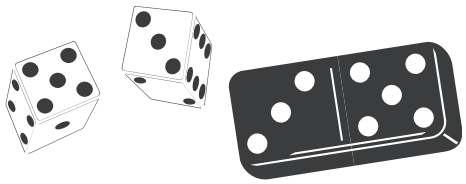
'What are mental strategies or techniques?'

There are several explanations, but I like the definition offered by McIntosh, Reys and Reys (1997) who described the purpose of mental or thinking strategy as turning a 'calculation we cannot do into a calculation we can do by employing relationships between numbers and operations'. (p. 323)

Subitising

Before discussing the development of mental strategies it would be appropriate to comment on children's ability to subitise. Subitising is a technical term that comes form the Latin root *subito*, meaning suddenly or immediately. Basically it means you have the ability to glance at a small group of objects and know how many are in that group. For example, you might look at the dots on one face of a die and realise that three dots are showing without needing to count the individual dots.

Dominoes and dot dice may be used to enhance this ability. Dominoes have the advantage of coming in double nine, double twelve and even double fifteen configurations. A simple game for developing this instant recognition involves rolling two dot dice and then matching them to the equivalent domino piece. For example, if a 5 and a 3 are rolled, the child would need to look for the corresponding 5, 3 domino piece.



Later, a numeral die may be substituted to encourage the matching of numerals with dots. Eventually two numeral dice may be used. (For further ideas for using dice and dominoes see *Dice Dilemmas*, *Domino Deductions* and *Dice Dazzlers*.) See 'Where can I find out more? 'p15.

Counting Strategies

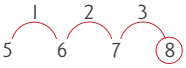
Young children primarily rely on counting strategies. For example, when adding 3 and 5 some children will count 1, 2, 3, 4, 5, 6, 7, 8 using their fingers to keep track. Other children will use a more sophisticated strategy, counting 3, 4, 5, 6, 7, 8. An even more sophisticated strategy would involve counting on from the larger number 5, 6, 7, 8. This strategy relies on the child understanding that the result is the same even if you change the order of the addition. Some children may become overly reliant on counting strategies which can cause them difficulties later. For example, a student using a counting on strategy to add 28 and 27 would likely lose count or miscount. Another problem is that children can over-generalise a particular strategy. In the above addition example the commutative property of addition was applied; that is, 3 + 5 will produce the same result as 5 + 3. However, this property does not apply in the case of subtraction, where 5 – 3 is not the same as 3 – 5. Examples of children employing this incorrect thinking may be seen when children start performing written calculations. When confronted with a question like 243 – 125, rather than rename the 243 to account for the fact that five ones can not be subtracted from

E X A M P L E

Counting On

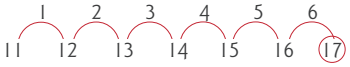
3 + 5 =

(change order) 5 + 3 =



6 + 11 =

11 + 6 =



Doubling

3 + 3 = 6

5 + 5 = 10

13 + 13 = 26

Students use these known number facts to derive near doubles

Near Doubles

Known number fact → 5 + 6 = (5 + 5) + 1 = 10 + 1 = 11

OR

Known number fact → 5 + 6 = (6 + 6) – 1 = 12 – 1 = 11

Bridging a Ten

Known number fact → 9 + 6 = (9 + 1) + 5 = 10 + 5 = 15

OR

Known number fact → 7 + 5 = (7 + 3) + 2 = 10 + 2 = 12

Partitioning

Known number fact → 5 + 8 = (5 + 5) + 3 = 10 + 3 = 13

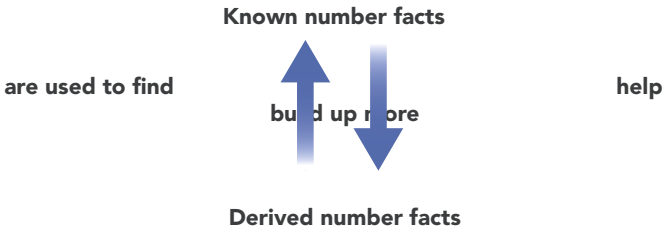
OR

Known number fact → 9 + 8 = (5 + 4) + (5 + 3) = (5 + 5) + (4 + 3) = 10 + 7 = 17

three ones, the child simply swaps the numbers, effectively saying 'I can't do three take five but I can do five take three'. This is a mistake that teachers commonly note.

As students' thinking develops they start to adopt more sophisticated strategies, such as doubling, or the use of near doubles. Bridging a ten is another important strategy that children develop to ease the burden of mental calculation. For example, when adding nine and six it is simpler to add one to nine to make ten and then add ten and five. This strategy relies upon the ability of a child to *partition* or split numbers up. This strategy is extremely important as it is the basis behind many other strategies. For example, in later years when required to add 25 and 27 you may choose to partition 27 into 25 + 2 so that you can add 25 and 25 and then 2 to reach an answer of 52. Alternatively you could add 25 and 5 and then 22. Of course you may have used an alternative strategy such as doubling to reach the same answer.

Eventually, the basic addition and subtraction facts become known and may be used to derive other facts. A child might know that 8 and 4 make 12 and use this piece of knowledge to work out the result of adding 8 and 5. The strategy of using a known fact to derive the answer to an unknown question relies on developing a bank of unknown facts. Mike Askew (1998) uses the following diagram to explain how this relationship works.



Eventually, derived number facts become known number facts and, in turn, as the range of known number facts expands, so do the opportunities children have for deriving facts.

Eventually children develop fluency with the basic addition and related subtraction facts. Basic addition facts run to 9 + 9 and the associated subtraction facts from 18 – 9. It is not until much later that children are ready to learn the multiplication facts or 'tables' as they are commonly referred to.

What can I do to build up children's mental strategies?

Basically, give children opportunities to discuss and share their mental methods in a supportive environment. A rapid-fire twenty questions a the start of a lesson does not help a child to develop mental strategies. Likewise, there is evidence to suggest that teaching children standard written algorithms too early may cause them to abandon sound mental strategies and adopt mental versions of the written algorithm. In effect, the children become worse at mental computation.

Where can I find out more?

I recommend reading:

McIntosh, A., DeNardi, E., Swan P. (1994). Think mathematically: How to teach mental maths in the primary classroom. Longman Cheshire

For dice and domino games see:

Swan P. (1997) Dice Dilemmas: Activities to promote mental computation and develop thinking about chance processes. Perth: A-Z Type.

Swan P. (2001) Domino Deductions: Developing mathematics from dominoes. Perth: A-Z Type.

Swan P. (2003) Dice Dazzlers: Short and simple dice games to promote numeracy. Perth: A-Z Type.

References

Askew M. (1998) Teaching Primary mathematics: A guide for newly qualified and student teachers. Great Britain: Hodder & Stoughton.

McIntosh, A. Reys, R. & Reys, B. (1997) Mental computation in the Middle Grades: The importance of Thinking Strategies. Mathematics in the Middle School, 2(5), 322-327.

INreview

Art of Recycling

Author:

Hilary Ansell

Features:

Reinforces the concept of recycling through practical ideas and the use of recycled materials. Suitable for a wide range of age groups at the primary school level but will need to be adapted to suit younger students' needs. Easy to follow instructions and excellent pictorial examples.

Suggestions for use:

Great card ideas, gifts and theme activities; for example, older students studying different ancient cultures (Roman, Greek) or artists (Van Gogh, Monet) or younger students studying different cultures around the world, musical instruments, jewellery or fun gifts. The projects cover a wide range of art/craft techniques and skills and the completed work can make wonderful classroom displays.

Fox

Author:

Margaret Wild

Illustrator:

Ron Brooks

Features:

This is a moving story about two friends, Dog and Magpie. When Fox comes along, Magpie must choose to stay with his loyal friend Dog or go with Fox. It is a story about friendship, loyalty and betrayal.

The illustrations are innovative and powerful with their layers of different mediums, use of colour and scratchy lines.

Suggestions for use:

This book can be used to discuss any of the issues of friendship, belonging, loyalty and betrayal.

Some activity ideas:

- Discuss the Australian Outback where the three animals live and locate on a map of Australia. Look in magazines and books for pictures similar to where Magpie, Dog and Fox lived.
- Discuss introduced species.
- Colour a piece of paper in different coloured wax crayons, completely covering the page. Add some detergent to acrylic black paint and paint over the top of the crayon picture. With a wooden skewer, scratch out a picture in the black paint when it is dry.

Multicultural Crafts

Authors:

Deborah Whitacre and Becky Radtke

Features:

Suitable for pre-primary to Year 4. Uses everyday 'junk' materials to create interesting crafts with a difference and an educational purpose. Clearly set out, easy to follow instructions and diagrams with reproducible patterns where needed.

Suggestions for use:

Compliments the study of different cultures around the world or a recycling theme.